Greedy approach to investigating cyclic structure of Cayley graphs

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Outline of the talk

The main goal

To overview recent results on greedy approach with emphasizing on ways of constructing (hamiltonian) cycles in Cayley graphs.

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Content

◇ Hamiltonian problem:
 - graphs
 - Cayley graphs

◇ Greedy approach:
 - constructing hamiltonian cycles
 - constructing non-hamiltonian cycles

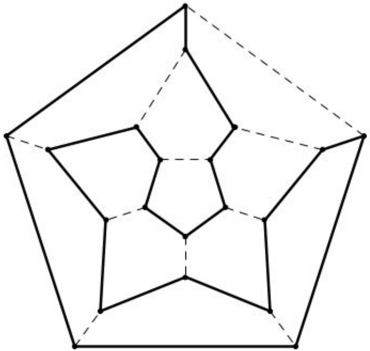
◇ Cyclic coverings and algebraic approach

◇ Open problems

Hamiltonicity of graphs

Hamiltonian graphs

Let $\Gamma = (V, E)$ be a connected graph where $V = \{v_1, v_2, \ldots, v_n\}$.
A *Hamiltonian cycle* in Γ is a spanning cycle $(v_1, v_2, \ldots, v_n, v_1)$.
A *Hamiltonian path* in Γ is a path $(v_1, v_2, \ldots, v_n)$.
A graph is *Hamiltonian* if it contains a Hamiltonian cycle.



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Hamiltonian problem, 1850s

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NP–completeness, 1979

Testing whether a graph is Hamiltonian is an NP-complete problem. [M.R. Garey, D.S. Johnson, Computers and intractability. A quide to the theory of NP–completeness].

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Applications

Hamiltonian paths and cycles naturally arise in:

- computer science
- word-hyperbolic groups and automatic groups
- combinatorial designs
- combinatorial optimization (travelling salesman problem)

Hamiltonicity of vertex-transitive graphs: Lovász conjecture, 1970

There is a famous Hamiltonicity problem for vertex–transitive graphs which was posed by László Lovász in 1970 and well–known as follows.

Question

Does every connected vertex–transitive graph with more than two vertices have a Hamiltonian path?

To be more precisely he stated a research problem asking how one can

> *" ... construct a finite connected undirected graph which is symmetric and has no simple path containing all the vertices. A graph is symmetric if for any two vertices x and y it has an automorphism mapping x onto y."*

However, traditionally the problem is formulated in the positive and considered as the Lovász conjecture that every vertex–transitive graph has a Hamiltonian path.

Hamiltonicity of vertex-transitive graphs: Lovász conjecture vs Babai conjecture

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For some $\varepsilon > o$, there exist infinitely many connected vertex–transitive graphs (even Cayley graphs) Γ without cycles of length $\geqslant (1-\varepsilon)|V(\Gamma)|$.

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A step forward in Lovász conjecture was made recently.

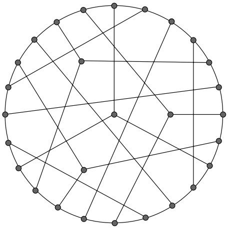
S. Du, K. Kutnar, D. Marusic, 2018

With the exception of the Petersen graph, a connected vertex-transitive graph of order pq, where p and q are primes, contains a Hamiltonian cycle.

Hamiltonicity of Cayley graphs: folk conjecture

There are only 4 vertex–transitive (not Cayley) graphs which do not have a Hamiltonian cycle, and have a Hamiltonian path:

- Petersen graph
- Coxeter graph
- two graphs obtained from the graphs above by replacing each vertex with a triangle and joining the vertices in a natural way



Conjecture on Cayley graphs

Every connected Cayley graph on a finite group has a Hamiltonian cycle.

Hamiltonicity of Cayley graphs: positive answers

D. Marušič, 1983

A Cayley graph $\Gamma = Cay(G, S)$ of an abelian group G with at least three vertices contains a Hamiltonian cycle.

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I. Pak, R. Radoičić, 2009

Every finite group G of size $|G| \geqslant 3$ has a generating set S of size $|S| \leqslant \log_2 |G|$ such that the corresponding Cayley graph $\Gamma = Cay(G, S)$ has a Hamiltonian cycle.

Hamiltonicity of Cayley graphs: positive answers

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This result has been generalized as follows.

M. Tchuente, Generation of permutations by graphical exchanges, 1982

Let S be a set of transpositions that generate Sym_n. Then there is a Hamiltonian path in the graph $Cay(Sym_n, S)$ joining any permutations of opposite parity.

Thus, all transposition Cayley graphs are Hamiltonian.

Greedy approach

A greedy algorithm

A greedy algorithm is an algorithmic paradigm that follows the problem solving heuristic of making the locally optimal choice at each stage with the hope of finding a global optimum.

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At each stage visit an unvisited city nearest to the current city

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In mathematical optimization, greedy algorithms solve combinatorial problems having the properties of matroids.

Greedy generation of permutations

A. Williams, J. Sawada, Greedy pancake flipping (2013)

Take a stack of pancakes, numbered $1, 2, \ldots, n$ by increasing diameter, and repeat the following:

Flip the maximum number of topmost pancakes that gives a new stack.

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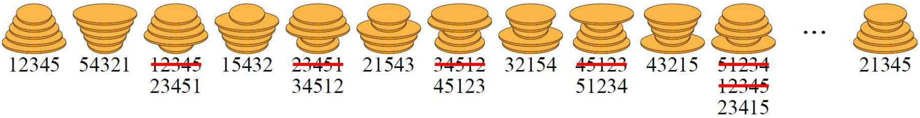

$[\overline{1234}]$ $[\overline{432}1]$ $[\overline{2341}]$ $[\overline{143}2]$ $[\overline{3412}]$ $[\overline{214}3]$ $[\overline{4123}]$ $[\overline{32}14]$

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is the Cayley graph on the symmetric group Sym_n with generating set $\{r_i \in Sym_n, 1 \leqslant i < n\}$, where r_i reverses the order of any substring $[1, i]$, $1 < i \leqslant n$, of a permutation π when multiplied on the right, i.e., $[\pi_1 \ldots \pi_i \pi_{i+1} \ldots \pi_n] r_i = [\pi_i \ldots \pi_1 \pi_{i+1} \ldots \pi_n]$.

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Williams' prefix–reversal Gray code: $r_n \, r_{n-1} \, r_{n-2}, \ldots, r_3, r_2$

Flip the maximum number of topmost pancakes that gives a new stack.

Zaks' (1984) prefix–reversal Gray code: $r_2 \, r_3, \ldots, r_{n-2} \, r_{n-1} \, r_n$

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Gray codes: generating permutations

V.L. Kompel'makher, V.A. Liskovets, Successive generation of permutations by means of a transposition basis, 1975

Q: Is it possible to arrange permutations of a given length so that each permutation is obtained from the previous one by a transposition?
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ryjStart with $I_n = [12 \dots n]$ and in each step reverse a certain suffix. Let ζ_n is the sequence of sizes of these suffixes defined by recursively as follows:

$$\zeta_2 = 2$$
$$\zeta_n = (\zeta_{n-1}\, n)^{n-1}\, \zeta_{n-1},\ n > 2,$$

where a sequence is written as a concatenation of its elements.

Zaks' algorithm: examples

If $n = 2$ then $\zeta_2 = 2$ and we have:

$$[\underline{12}] \quad [21]$$

If $n = 3$ then $\zeta_3 = 23232$ and we have:

$$[1\underline{23}] \quad [2\underline{31}] \quad [3\underline{12}]$$

$$[\underline{132}] \quad [\underline{213}] \quad [321]$$

If $n = 4$ then $\zeta_4 = 23232423232423232423232$ and we have:

$$[12\underline{34}] \quad [23\underline{41}] \quad [34\underline{12}] \quad [41\underline{23}]$$

$$[1\underline{243}] \quad [2\underline{314}] \quad [3\underline{421}] \quad [4\underline{132}]$$

$$[13\underline{42}] \quad [24\underline{13}] \quad [31\underline{24}] \quad [42\underline{31}]$$

$$[1\underline{324}] \quad [2\underline{431}] \quad [3\underline{142}] \quad [4\underline{213}]$$

$$[14\underline{23}] \quad [21\underline{34}] \quad [32\underline{41}] \quad [43\underline{12}]$$

$$[\underline{1432}] \quad [\underline{2143}] \quad [\underline{3214}] \quad [4321]$$

Greedy hamiltonian cycles in the Pancake graph

Greedy approach for constructing greedy cycles

Consider a sequence $GP = (r_{m_1}, r_{m_2}, \ldots, r_{m_k})$ of distinct $k \leqslant n-1$ prefix–reversals $r_{m_j}, 2 \leqslant m_j \leqslant n$, from the generating set of P_n.

A greedy cycle is formed by consecutive application of the leftmost suitable prefix–reversal from GP which is called a greedy sequence of length k in this setting.

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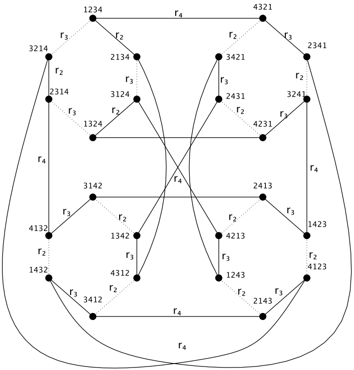
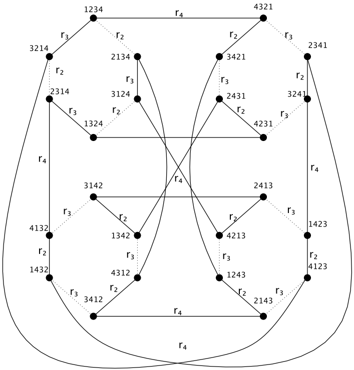
Known greedy sequences for the Pancake graph

Sawada-Williams' sequence: $(r_n, r_{n-1}, \ldots, r_3, r_2)$ (2013)
Zaks' seguence: $(r_2, r_3, \ldots, r_{n-1}, r_n)$ (1984)
K-Medvedev' sequences: $(r_n, r_{n-1}, \ldots, r_2, r_3)$, $(r_3, r_2, \ldots, r_{n-1}, r_n)$ (2016)

Example: greedy hamiltonian cycles in P_4



(r_4, r_3, r_2)-greedy cycle



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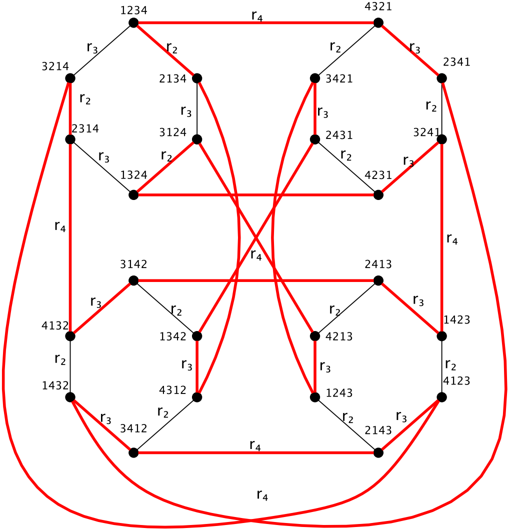

Are there other greedy sequences in P_n?

Results of the numerical experiment

Index of $\mathbf{P_n}$	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}
# of all possible GP	24	120	720	5040	40320	9!	10!
# of GP with proper length	6	16	20	76	162	456	846

None of the obtained sequences were Hamiltonian.

EK, A.N. Medvedev, 2016

Suppose H_n^G is a greedy Hamiltonian cycle in the $P_n, n \geqslant 4$, with the $GP = (r_{m_1}, r_{m_2}, \ldots, r_{m_k})$, $k \leqslant n-1$. Then the length of H_n^G satisfies

$$|H_n^G| = n! = \frac{1}{2^{k-2}} \prod_{i=1}^{k-1} l_i,$$

where l_i is the length of a cycle of form $C_{l_i} = (r_{m_i} r_{m_{i+1}})^{k_i}, 2 \leqslant i \leqslant k$.

Independent cycles in P_n

EK, A.N. Medvedev, 2016

The Pancake graph $P_n, n \geqslant 4$, contains the maximal set of $\frac{n!}{\ell}$ independent ℓ–cycles of the canonical form

$$C_\ell = (r_n\, r_m)^k, \tag{1}$$

where $\ell = 2\,k$, $2 \leqslant m \leqslant n-1$ and

$$k = \begin{cases} O(1) & \text{if} \quad m \leqslant \lfloor \frac{n}{2} \rfloor; \\ O(n) & \text{if} \quad m > \lfloor \frac{n}{2} \rfloor \quad \text{and} \quad n \equiv 0 \pmod{n-m}; \\ O(n^2) & \text{else.} \end{cases} \tag{2}$$

The cycles presented in Theorem have no chords.

Greedy hamiltonian cycles

General question

Are there greedy hamiltonian cycles in other Cayley graphs?

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Question

Are there greedy hamiltonian cycles in the Star graphs?

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Are there greedy hamiltonian cycles in the Bubble-Sort graphs?

Star graphs: definition

The Star graph $S_n = Cay(Sym_n, T),\ n \geqslant 2$

is the Cayley graph on the symmetric group Sym_n of permutations $\pi = [\pi_1\pi_2 ... \pi_i ... \pi_n]$ with the generating set T of all transpositions $t_i = (1\ i)$ swapping the 1st and ith elements of a permutation π.

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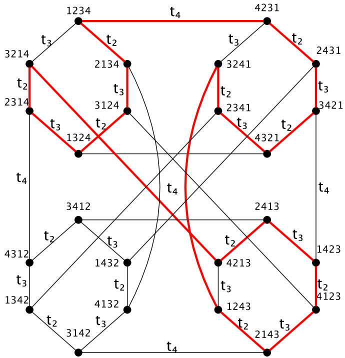
Properties of the Star graph

- connected bipartite $(n-1)$–regular graph of order $n!$ and diameter $diam(S_n) = \lfloor \frac{3(n-1)}{2} \rfloor$ (S. B. Akers, B. Krishnamurthy, 1989)
- vertex-transitive and edge-transitive
- contains hamiltonian cycles (V. Kompel'makher, V. Liskovets, 1975, P. Slater, 1978)
- it does contain even ℓ–cycles where $\ell = 6, 8, \ldots, n!$
- has integral spectrum

Example: is (t_2, t_3, t_4) a greedy sequence in S_4?



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Greedy cycles in the Star graphs

Theorem *(D. Gostevsky, EK, 2018)*

In the Star graph $S_n, n \geqslant 3$, any greedy sequence GS of length k, where $2 \leqslant k \leqslant n-1$, forms a GS-greedy cycle of length $2 \cdot 3^{k-1}$.

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Proof

If $n = 3$, then $S_3 \cong C_6$, hence $GS_3 = (t_2, t_3)$ is a greedy sequence generating six permutations as follows:

$$GS_3 : \qquad [123] \xrightarrow{t_2} [213] \xrightarrow{t_3} [312] \xrightarrow{t_2} [132] \xrightarrow{t_3} [231] \xrightarrow{t_2} [321],$$

which obviously forms a cycle of length $2 \cdot 3^{2-1} = 6$.

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Proof

If $n = 4$, then $GS_4 = (t_2, t_3, t_4)$ forms a greedy cycle of length $6 \cdot 3 = 2 \cdot 3^{3-1} = 18$ in S_4:

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Corollary

There are no greedy hamiltonian cycles in S_n for $n \geqslant 4$.

Greedy cyclic covering in the Star graphs

Let $\mathfrak{F} = \{GS_k = (t_2, t_3, \ldots, t_k),\ 3 \leqslant k \leqslant n\}$ be a family of greedy sequences.

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Theorem *(D. Gostevsky, EK, 2017)*

In the Star graph S_n, $n \geqslant 3$, there exists a maximal set of independent cycles formed by greedy sequences from the family $\mathfrak{F}$ consisting of the following cycles:

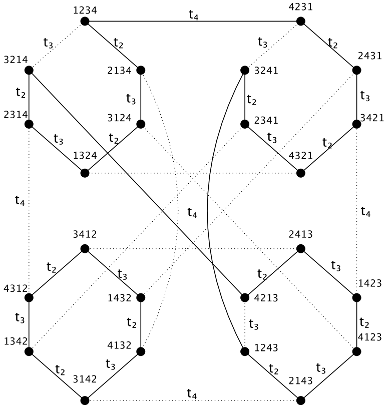
(1) one cycle of length $2 \cdot 3^{n-2}$, and

(2) $n-3$ cycles of length $2 \cdot 3^{n-3}$ when $n \geqslant 4$, and

(3) N_m cycles of length $2 \cdot 3^{n-m-2}$ for all $2 \leqslant m \leqslant n-3$ when $n \geqslant 5$, where

$$N_m = \left(\prod_{l=2}^{m} (n-l+2) \right) \cdot (n-m-2).$$

Example: GS-greedy cyclic covering of S_4



Independent greedy 18- and 6-cycles are formed by greedy sequences $GS_4 = (t_2,\ t_3,\ t_4)$ and $GS_3 = (t_2, t_3)$.

Algebraic approach: to be continuied...

From cycle covering to hamiltonian cycle: idea

◇ - find cycle coverings in a graph

◇ - use algebraic operations on cycle coverings to get a hamiltonian cycle

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The technique of creating large cycles from the symmetric difference of small cycles has been used by change ringers for hundreds of years [R. Duckworth and F. Stedman, Tintinnalogia, Self-published, 1667 (The Art of Ringing)].

Algebraic approach: to be continuied...

From cycle covering to hamiltonian cycle: idea

◇ - find cycle coverings in a graph
◇ - use algebraic operations on cycle coverings to get a hamiltonian cycle

The technique of creating large cycles from the symmetric difference of small cycles has been used by change ringers for hundreds of years [R. Duckworth and F. Stedman, Tintinnalogia, Self-published, 1667 (The Art of Ringing)].

Thanks for attention!